Rheological Modeling of Plug-Assist Thermoforming

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ABSTRACT: Solving problems for thermoforming processes in the production of axisymmetric thin walled plastics is investigated in this research work. A nonlinear viscoelastic rheological model with a new strain energy function is suggested for improvement of physical properties of final product. For model validation, a quantitative relation between stress and technical parameters of plug-assist thermoforming is determined by comparison of theoretical and experimental results. This process with the proposed rheological model could be suggested for prevention from some technical defects such as wall thickness variations, physical instability during inflation-shrinkage, and warpage exhibited in the final part of a polymeric sheet thermoforming.
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Key words: plug-assisted thermoforming; rheological modeling; nonlinear viscoelastic model; thermoforming defects

INTRODUCTION

The polymer processing for production of all forms of polymeric articles has found a great place in chemical industries. Thermoforming process is one of the most popular techniques in this field. It applies to thermoplastic sheet or film-forming techniques for various packaging applications such as medical devices, electronic instruments, food containers, and pharmaceuticals. Wide applications of thermoforming process are due to its high performance, simplicity, compactness, and relatively low-cost equipment. These issues make it possible to produce complex, large-scale configurations and any shapes of products. As there are many parameters that affect the thermoforming process, it becomes very difficult to optimize processing conditions.[1–3] A thermoforming process includes the following steps: clamping, heating in the range of $T_g < T < T_m$, shaping under pressure, cooling, and trimming. There are many ways to stretch sheets: vacuum, air pressure, and mechanical aids such as implementation of a plug. For increasing the quality of products such as narrow wall-thickness tolerance or elimination of frozen-in stresses, a combination of mechanical and vacuum or pressure forming methods may be implemented. In early step, it involves the usage of mechanical prestretching with plug and then vacuum or pressure forming is applied. Figure 1 illustrates this process.

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Deformation process in the stage of mechanical prestretching has a great influence on the operational and exploitation properties of polymeric products. To optimize the processing conditions and increase the quality of products, it is necessary to evaluate the parameters that are varying during the deformation process. This is possible only through mathematical description of the deformation process. However, one of the major involving issues is the choice of a correct rheological model that has the ability of description of a polymers stress–strain behavior versus kinematical conditions of deformation.[4]

Many dissertations and research papers had been published on presentation of this theory,[4–6] but most of them have an important deficiency. They are merely based on elastic behavior of deformed sheets without taking into account the viscous properties of materials. It is well known that good thermoforming materials must have suitable viscous characteristics to provide a certain polymer flow under the applied stress. The sufficient elastic characteristics avoid excessive thinning.[7,8] Generally, dynamic viscoelastic models are required for accurate prediction of the elongational behavior including elongational viscosity and strain hardening and/or softening. The model parameters can be set by experiments. Moreover, deformation of polymers in the process of inflation may be transient, which makes the model in a time varying differential form. Also, the choice of the rheological model is important not only for mathematical representation of deformation processes in production of hollow polymeric articles, but also it may be used to reduce the defects of produced products such as shrinkage.

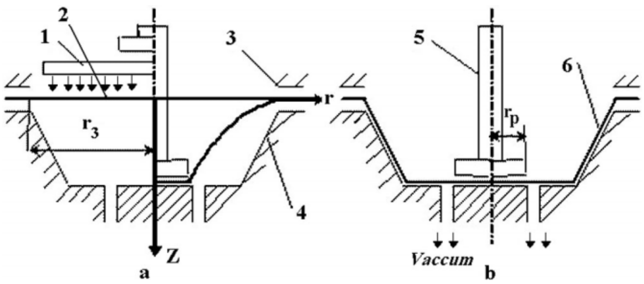

Figure 1 Flow diagram of plug-assisted thermoforming (a: heater and mechanical prestretching; b: vacuum thermoforming). 1: Heater; 2: sheet prior to forming; 3: clamp frame; 4: mold; 5: plug; 6: deformed sheet.

RHEOLOGICAL MODEL

One of the most practical problems facing thermoforming is to predict shrinkage and warpage. Shrinkage and warpage prediction is very important because of processing constraints and finally may cause system failure. Unfortunately, this problem is an overlooked area in thermoforming research. Successful solution to it directly depends on the right selection of a proper rheological model.

One of the theoretical models in this area was developed by Leonov[9]:

$$\sigma + p\delta = 2cW_1 - 2c^{-1}W_2$$

$$e_f = 1/\theta_0 G_0(T)\exp\{-\beta\, w^s/G_0(T)\} \times [(c - I_1\delta/3)W_1^s - (c^{-1} - I_2\delta/3)\, W_2^s] \quad (1)$$

$$\frac{dc}{dt} + \omega c - c\omega - c(e - e_f) - (e - e_f)c = 0$$

where σ is the stress tensor, p is the Lagrange multiplier, determined by boundary condition, δ is the identity tensor, c is the Cauchy strain tensor, e_f is the flow strain rate tensor, ω is the vortex tensor, e is the strain rate tensor, $\theta_0(T)$ is the relaxation time, $G_0(T)$ is the tensile modulus, W is the strain energy function $W = W(I_1,I_2)$, β is flexibility parameter of macromolecular chains, I_1 and I_2 are the primary and secondary strain tensor invariants, and $2W^S = W(I_1, I_2) + W(I_2, I_1)$ the symmetric function of W.

The last one can be shown by:

$$W_1 = \frac{\partial W}{\partial I_1},\ W_2 = \frac{\partial W}{\partial I_2}$$

$$W_1^S = \frac{\partial W^S}{\partial I_1},\ W_2^S = \frac{\partial W^S}{\partial I_2}.$$

STRAIN ENERGY FUNCTION MEASUREMENT

But in practice, there is a problem for application of eq. (1). This problem arises from the choice of strain energy function $W = W(I_1, I_2)$. Most researchers use Mooney-Rivlin potential, but there are differences between experimental and theoretical results for prediction of stress and strain. Results of recent research show that in various kinematical deformations, the following potential can be used[10,11]:

$$W = 0.25G_0(I_1 + I_2 - 6). \quad (2)$$

Now, by utilizing eq. (1) with regards to strain energy function (2), a mathematical description for deformation process in mechanical prestretching thermoforming can be developed [Fig. 1(a)].

MEASUREMENT OF RHEOLOGICAL AND MATERIAL PARAMETERS

Considering axisymmetric articles, when initial sheet with radius (r_3) is heated, it is deformed because of plug movement with radius of r_p at constant velocity V_p, in direction of z axis. The implemented material is assumed to be incompressible and isotropic. The deformation process is carried out under isothermal condition. The deformed sheet could be considered a thin-walled shell, thus, the hot polymer can be modeled as a membrane. Therefore, the bending resistance of the hot sheet is ignored and the material thickness is assumed to be small in comparison to dimensions of the material. Three different stretch ratios are involved in deformation process:

$$\lambda_1 = \frac{d\xi}{d\xi_0};\ \lambda_2 = \frac{r}{r_0};\ \lambda_3 = \frac{h}{h_0} \quad (3)$$

where λ_1, λ_2, and λ_3 are the principal stretch ratios in the longitudinal, azimuthal, and thickness directions of the membrane respectively. They are related together by the incompressibility condition $\lambda_1\ \lambda_2\ \lambda_3 = 1$ and ξ and ξ_0 are length of longitudinal in deformed and strainless sheet.

r and r_0 are the radii of deformed and strainless sheet, respectively and h and h_0 are the thicknesses of the sheet after and before deformation respectively.

Mechanical prestretching is a planar stretching (pure shear). Therefore, the following conditions exist:

$$\lambda_2 = 1;\ \sigma_3 = 0;\ \lambda_3 = \lambda_1^{-1} \quad (4)$$

With respect to the conditions and the tensors in eq. (1), the following expression can be written:

$$e = \dot{\varepsilon}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix};\ \omega = 0;$$

$$\mathrm{c} = \begin{pmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c^{-1} \end{pmatrix}; \mathrm{c}^{-1} = \begin{pmatrix} c^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{pmatrix} \quad (5)$$

where $\dot{\varepsilon}$ is the rate of deformation in longitudinal direction. The primary and secondary invariants of tensor c are resulted from eq. (5) as:

$$I_1 = I_2 = c + I + c^{-1} \quad (6)$$

By utilizing eqs. (2), (5), and (6), the following form of eq. (1) can be developed.

$$\sigma + p\delta = 0.5G_0(T) \begin{pmatrix} c - c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1} - c \end{pmatrix} \quad (7)$$

$$\mathrm{e}_f = -\frac{1}{4\theta_0(T)} \exp[-\beta(c + c^{-1} - 2)] \times \begin{pmatrix} c - c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1} - c \end{pmatrix} \quad (8)$$

Parameter p is resulted from condition (4): $\sigma_3 = 0$. By substituting expressions (5) and (8) into eq. (1):

$$\frac{dc}{dt} = 2\left[\dot{\varepsilon}c - (c^2 - 1) - \frac{1}{4\theta_0(T)} \exp[-\beta(c + c^{-1} - 2)]\right] \quad (9)$$

This differential equation defines kinetics of elastic strain during the deformation process of viscoelastic media. The deformation rate is defined as follows:

$$\dot{\varepsilon} = \frac{d\varepsilon^H}{dt} = \frac{d\ \ln\lambda}{dt} = \frac{1}{\lambda}\frac{d\lambda}{dt} \quad (10)$$

where ε^H is Hencky strain.

From relations (3) and (4), there are

$$\lambda_1{}^2\ (\tilde{z}) \equiv \lambda^2(\tilde{z}) = 1 + \left(\frac{d\tilde{z}}{d\tilde{r}}\right)^2 \quad (11)$$

where $\tilde{z} \equiv \frac{z}{r_3}$ and $\tilde{r} \equiv \frac{r}{r_3}$ are dimensionless axes.

The relation $\tilde{r} = f(\tilde{z})$ can be defined by applying the following equation[6]:

$$\lambda^2(\tilde{z}) = 1 + \left(4\sqrt[4]{b} + \frac{\tilde{z}}{4\sqrt[4]{b}}\right)^4 \quad (12)$$

where $b \equiv \lambda^2(\tilde{z} = 0) - 1$.

By combining expressions (11) and (12), a differential equation with initial condition ($\tilde{z} = 0$) is resulted:

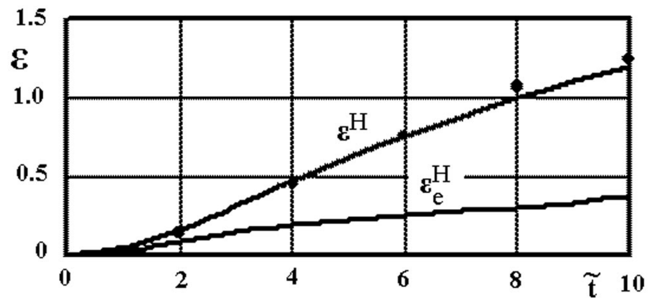

Figure 2 Kinetics of development of total (ε^H) and elastic (ε_e^H) deformation in mechanical prestretching: line, analytic solution; points, experimental data[4] ($a = 3.16$; $\theta_0 = 0.3$ s).

$$\tilde{r}(\tilde{z}) = \frac{\sqrt{b}}{\sqrt{b + \tilde{z}}} \quad (13)$$

During mechanical prestretching, plug is moved with constant velocity V_p. Therefore: $z = V_p t$.

Using this condition, the eq. (13) converts to

$$b \equiv \lambda^2(0) - 1 = \left(\frac{V_p t}{r_3}\frac{\tilde{r}_p}{1 - \tilde{r}_p}\right)^2 \quad (14)$$

From eq. (10) by using eqs. (12)–(14), the following form is obtained.

$$\dot{\varepsilon}(t) = \frac{t}{t^2 + \tilde{r}^4\left[\frac{r_3}{V_p}\left(\frac{1 - \tilde{r}_p}{\tilde{r}_p}\right)\right]^2} \quad (15)$$

The earlier equation may be represented in a dimensionless state form:

$$E(\tilde{t}) \equiv \dot{\varepsilon}(t)\theta_0(T) = \frac{\tilde{t}}{\tilde{t}^2 + a^2} \quad (16)$$

Where $\tilde{t} \equiv \frac{t}{\theta_0(T)}$; $a \equiv \tilde{r}^2 \frac{r_3}{V_p\theta_0(T)} \frac{1 - \tilde{r}_p}{\tilde{r}_p}$

And finally, from the eq. (7) and condition (4), there is:

$$\sigma_1 \equiv \sigma = G_0(T)\ (c - c^{-1}) \quad (17)$$

RESULTS AND DISCUSSION

By using differential eqs. (9) and (16), unknown dependences that describe the deformation process in polymeric sheets are found (Fig. 2).

Analysis of data shows that mechanical prestretching results in both nonhomogeneity and nonstationary deformation that affects the used sheet, $\varepsilon^H = f(\tilde{t},\tilde{r})$. Realization of nonhomogeneity and nonequilibrium conditions during elastic deformation is $(\varepsilon_e^H(\tilde{t},\tilde{r}) = \ln\lambda_e(\tilde{t},\tilde{r}) = \ln\sqrt{c(\tilde{t},\tilde{r})}$. In conjunction with nonstationary viscous process of non-Newtonian, $\varepsilon_f^H = f(\tilde{t},\tilde{r})$ is determined simply by solving the second order differential equation of rheological model.[1] As results show, the kinetics of deformation process depends on dimensionless physical parameter of α. Thus, with modification of this parameter, deformation process can be gained. In other words, thermoformed articles with high performance may be produced by a proper setting of α value.

For instance, one of the requirements in the production of disposable polymeric food containers, such as plastic glasses, plates, and trays, is decreasing the possibility of warpage to a minimum during its contacts with hot food or during re-heating in microwave ovens, since warpage is due to relieving residual (frozen-in) stresses, which are accumulated during thermoforming process. This occurs in the stage of mechanical prestretching, vacuum-pressure forming, and cooling. Therefore, minimization of this defect is ensured by rationalization of parameter α. Validity of this result was confirmed not only qualitatively, but also quantitatively with performation of experiments on ABS sheets in temperature of 413 K that is exhibited in Figure 3.

As mentioned, since shrinkage and warpage are due to accumulation of stress, therefore, in this work for releasing of stresses, a pause ($\dot{\varepsilon} = 0$) is applied immediately after mechanical prestretching without removing the mechanical force (plug). This pause decreases the stress highly and subsequently a high reduction in shrinkage and warpage is gained (Fig. 4).

Parameter α depends on several factors such as velocity of plug, relation between plug and sheet radius, and thermodynamic state that defines velocity of relaxation process. The results also show that deformation process is not only mechanical, but also thermomechanical, because it depends on the temperature at which the process is realized. Figure 5 proves and confirms the earlier statement by presenting the coincidence of theoretical and experimental results. The values of theoretical parameters $G_0(T)$ and $\theta_0(T)$ for ABS sheet at different temperatures were resulted with the help of a program namely "Reocon."[10]

The last figure shows that temperature of sheet not only influences relaxation process causing the growth of elastic deformation, but also directly affects stress via tensile modulus. Thus, even in low elastic deformation, stress may be developed higher than sheet stability and so rupture in the sheet can be experienced. Some attention to this case has found in dissertations and papers.[6,11] It is a great technical defect. Since the reason for this defect is clear, it can be eliminated simply by setting the parameter of stretching sheet α. This is done by setting of applied stress within its stability limits of the implemented polymer in a fixed temperature under deformation process. By employing the developed equation as presented in this article, this problem can be solved easily.

As stated, one of the achievements of this work is the improvement in the quality of deformed sheet having varied wall-thickness. Dimensionless function of wall thickness distribution in mechanical prestretching is given by eqs. (3), (4), and (16) in the following form:

$$\lambda_3(\tilde{r}) \equiv \frac{h(\tilde{r})}{h_0} = \left[1 + \left(\frac{S}{r_3}\right)^2 \frac{\tilde{r}_p^2}{(1-\tilde{r}_p)^2}\frac{1}{\tilde{r}^4}\right]^{-0.5} \quad (18)$$

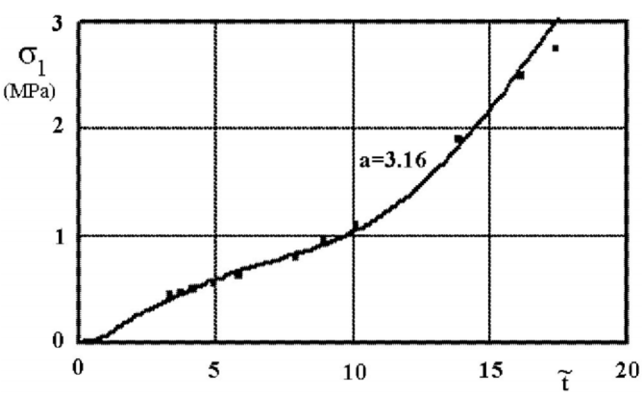

Figure 3 Kinetics of development of longitudinal stress; line, eq. (17); points, experimental data[4], $G_0 = 0.635$ MPa.

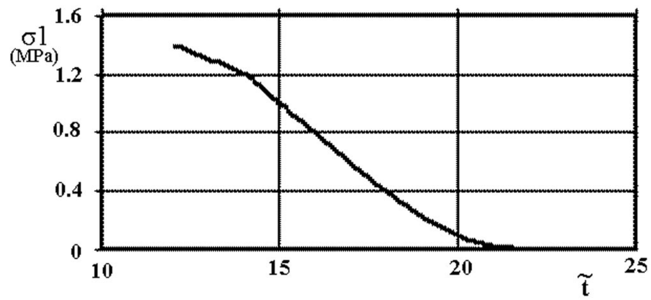

Figure 4 Stress relaxation after mechanical prestretching in $T = 413$ K, $a = 3.16$.

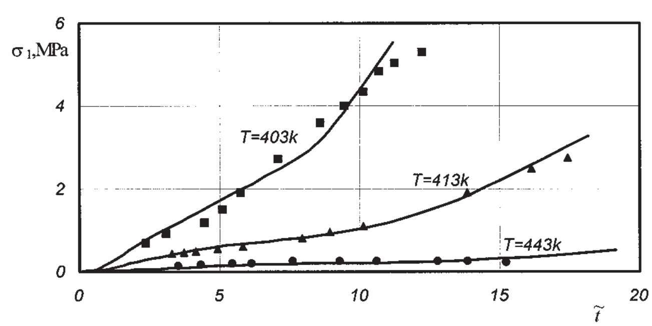

Figure 5.

where S is the depth of the deformed sheet.

Analysis of this equation shows that maximum and minimum wall-thickness are $\tilde{r} = I$ and $\tilde{r} = \tilde{r}_p$, respectively. In this manner, wall-thickness variation in deforming sheets is determined as follows:

$$\Delta \equiv I - \frac{h_{\min}}{h_{\max}} = I - \frac{\lambda_3(\tilde{r} = \tilde{r}_p)}{\lambda_3(\tilde{r} = I)} \quad (19)$$

Expressions (18) and (19) specify that wall-thickness variations do not depend on rheological properties of polymeric materials. The thickness variation is a function of dimensionless parameters such as the depth of deformed sheets and plug radius. It is decreased by increasing the plug radius and reducing the S value. Thus, by controlling the physical parameters such as the depth of the deformed sheet and plug radius, variation of wall-thickness is reduced.

It is clear that deformation process during mechanical prestretching influences second stage of thermoforming and consequently the quality of final products. Equations (13), (14), and (18) form initial condition for describing the second stage of thermoforming, pressure, or vacuum formation. It is essential for finding a solution to the problem.

Revealed results show that the stage of mechanical prestretching has a great effect on exploitation properties of final products. Optimum operation of the mentioned process by setting proper values of parameters in the implemented technology secures all of its requirements.

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